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OPEN-CHANNEL WAVES GENERATED BY PROPAGATION OF A DISCONTINUOUS WAVE OVER A BOTTOM STEP

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This paper presents the results of theoretical and experimental studies of open-channel waves generated by the propagation of a discontinuous dam-break wave over a bottom step. The cases where the initial tailwater level is higher than the step height (the step is under water) and where this value is smaller than the step height (at the initial time, water is absent on the step) are considered. Exact solutions are constructed using modified first-approximation equations of shallow-water theory, which admit the propagation of discontinuous waves in a dry channel. On the stationary hydraulic jump formed above the bottom step, the total free-stream energy is assumed to be conserved. These solutions agree with experimental data on various parameters (types of waves, wave propagation velocity, asymptotic depths behind the wave fronts).

Key words: *discontinuous wave, bottom step, shallow-water equations, experimental data.*

Introduction. The first-approximation equations of shallow-water theory [1–3] are widely used in modeling the propagation of discontinuous waves [4–7] (hydraulic bores [8, 9]) resulting from complete or partial break of hydraulic dams or the impact of large sea tsunami-type waves [10] on shallow water. However, the classical system of the basic conservation laws of shallow-water theory (consisting of the laws of conservation of mass and total momentum [3]), while correctly describing the parameters of discontinuous waves propagating in a liquid of finite depth above an even bottom [1], is not suitable for describing wave flows above various bottom-relief features, in particular, discontinuous-wave propagation over a bottom step or a drop. This is due to the fact that the total-momentum equation is an exact conservation law only in the case of a horizontal bottom and it cannot be used to derive Hugoniot conditions for the discontinuities arising from bed level changes.

A method of deriving relations for stationary discontinuities above bed level changes using the shallow-water equations was proposed in [11] and validated in [12]. This method is based on the assumption that if two characteristics arrive at such a discontinuity, then, along with the continuity of the discharge, which follows from the mass conservation law, it is necessary to require the continuity of the Bernoulli function, which follows from the local-momentum conservation law and the conservation law for the total free-stream energy. If three characteristics arrive at a discontinuity above a bed level change, the discharge continuity is sufficient to determine all flow parameters at this discontinuity. The total energy at such a discontinuity is lost, which serves as a criterion for its stability [3]. These assumptions and a generalized method of adiabats [13] were used to study the unique solvability of dam-break problems above a bottom step [14] and a drop [15]. The obtained self-similar solutions are in fairly good agreement with experimental data [16–18] on various parameters (types of waves, wave propagation velocity, asymptotic depths behind the wave fronts).

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The classical system of the basic conservation laws of shallow-water theory are not suitable for modeling discontinuous-wave propagation in a dry channel. The exact solutions describing water flow in a dry channel on the basis of these equations are continuous depression waves [1, 5]. This situation cannot be changed by accounting for bottom friction, which is a source term that makes no contribution to the Hugoniot conditions at the discontinuous-wave front [19, 20]. At the same time, numerous laboratory experiments have shown that these continuous solutions significantly overestimate the propagation velocity of the leading edge of the wave and distort its profile [21–24]. In experiments, the wave front propagating in a dry channel is much steeper and is subjected to breaking typical of discontinuous waves.

A method for modeling discontinuous-wave propagation in a dry channel using the shallow-water equations is proposed in [25]. This method is based on the modified total-momentum conservation law obtained in [26] in deriving shallow-water equations for the class of generalized solutions with discontinuous waves. This is done using a double passage to the limit that takes into account the turbulent viscosity effect inside the transition region corresponding to discontinuous waves. The use of this passage to the limit is an attempt to take into account the concentrated momentum loss due to the formation of local turbulent vortex structures in the liquid surface layer at a discontinuous wave front is undertaken. In [20], the modified shallow-water equations were employed to develop an implicit difference through-computation scheme that allows one- and two-dimensional (planned) calculations of discontinuous-wave propagation in a dry channel, runup of the waves on a coast, and flow over various coastal obstacles.

The present paper presents the results of theoretical and an experimental Studies of open-channel waves generated by the propagation of a discontinuous dam-break wave over a bottom step. In the experiments, the initial headwater level was constant and the tailwater level decreased gradually. As a result, in the first series of experiments, the initial tailwater level was higher than the step height (the step was under water), and in the second series, this value was lower than step height (at the initial time, water was absent on the step). For a uniform theoretical description of the results of these experiments, we used the modified system of the basic conservation laws of shallow-water theory proposed in [20, 26], which, on the one hand, admits discontinuous-wave propagation in a dry channel and, on the other hand, correctly reproduces the parameters of these waves for channels of finite depth. Exact solutions are constructed using the methods proposed in [14, 15] and the results of a study [27], in which the unique solvability of the problem for the step under water at the initial time was studied using the classical system of the basic conservation laws of shallow-water theory. A comparison with results of laboratory experiments was performed.

1. Experimental Technique. A diagram of the experiment, the main notation, and the coordinate system used below are presented in Fig. 1. The experiments were performed in a rectangular channel of width $b = 20.2$ cm and length $l_1 + l_2 + l_3 = 706.5$ cm, whose left part was in a tank of width $B = 100$ cm and length $l_4 = 330$ cm. The left open end of the channel was at a distance $l_5 = 130$ cm from the left end wall of the tank. The bottom of the channel consisted of two horizontal segments connected by a vertical step of height $\delta = 5.5$ cm located at a distance $l_3 = 238.5$ cm from the right end wall. The initial level difference $\Delta z = H - z_0$ was produced by a flat shield at a distance $l_2 = 122$ cm upstream of the bottom step. In the experiments, the headwater depth $H = 20.5$ cm was constant and the tailwater level changed from $z_0 = H$ to $z_0 = z_{\min} = 1$ cm. If $z_0 > \delta$, the water layer depth at the step was $H_0 = z_0 - \delta > 0$ and, the step was under water at the initial time, because of which the levels initial tailwater water ahead of and behind the step coincided. If $z_0 < \delta$, the step bottom was dry at the initial time.

The initial free-surface levels were determined by measuring needles with an absolute error not greater than 0.05 cm. At the time $t_0 = 0$, the shield was removes from the channel manually. The law of its motion was recorded by a rheochord transducer. The time taken to remove the shield did not exceed 0.05 sec. The removal of the shield, which modeled a dam-break event, resulted in the propagation of a discontinuous wave (shown by dots in Fig. 1) (hydraulic bore) in the positive x direction. After the initial discontinuous waves had passed over the step, secondary discontinuous waves formed which moved in the opposite direction. The propagation velocities of these waves and the asymptotic depths behind their fronts were measured in the experiments.

Free-surface level fluctuations were measured as functions of time t at specified points on the x longitudinal coordinate using wavemeters operating on the principle of difference in electric conductivities between water and air. The resolution of the wavemeters was 0.2 mm. The fluctuation frequency measured by the wavemeters with an error not greater than 10% did not exceed 10 Hz. Photo and video recording was also used. Below, the results

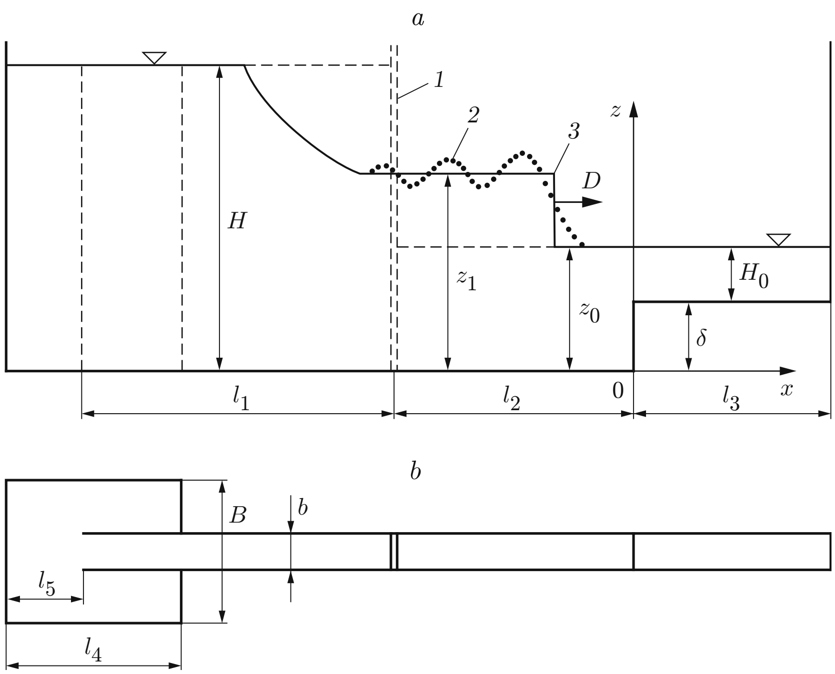

Fig. 1. Diagram of experiment: (a) side view; (b) top view; 1) initial position of the shield; 2) wave incident on the step after removal of the shield; 3) discontinuous wave modeling (based on the shallow-water equations) the incident wave.

of the experiments are given for the times at which the waves reflected from the right end wall of the channel and from the left end wall of the tank have not reached the channel cross section considered.

2. Formulation of the Problem Based on Shallow-Water Theory. In the case of a rectangular channel of constant width ignoring friction, the modified system of the basic conservation laws of shallow-water theory are written as [20, 26]

$$h_t + q_x = 0; \tag{2.1}$$

$$(q + \gamma u)_t + (qu + gh^2/2 + \gamma(u^2/2 + gh))_x = -g(h + \gamma)b_x, \tag{2.2}$$

where $h(x,t)$ is the flow depth, $q(x,t)$ is the specific discharge (per unit width of the channel), $u = q/h$ is the flow velocity, $z(x,t) = b(x) + h(x,t)$ is the free-surface level, $b(x)$ is the vertical coordinate of the bottom (bed level), g is the acceleration due to gravity and $\gamma = \gamma^* H$ is a dimensional parameter (H is the characteristic flow depth, which, for the problem considered, is set equal to the initial headwater depth; γ^* is a dimensionless parameter chosen so as to agree with the results of laboratory experiments [23]). Equation (2.1) is the law of conservation of mass, and equation (2.2) is the modified conservation law for the total momentum, which is derived in [26]. The classical conservation law for the total momentum

$$q_t + (qu + gh^2/2)_x = -ghb_x \tag{2.3}$$

follows from Eq. (2.2) for $\gamma = 0$. Formally, the modified total-momentum conservation law (2.2) is obtained by a linear combination of Eq. (2.3) and the equation

$$u_t + (u^2/2 + gz)_x = 0,$$

which is the local-momentum conservation law [3].

In contrast to the classical system (2.1), (2.3), the modified system (2.1), (2.2) admits the propagation of discontinuous waves in a dry channel. Its corresponding Hugoniot conditions above a horizontal bottom $b(x) = \text{const}$ are written as

$$D[h] = [q]; \tag{2.4}$$

$$D([q] + \gamma[u]) = [qu + gh^2/2] + \gamma[u^2/2 + gh], \tag{2.5}$$

where D is the propagation velocity of a discontinuous wave and $[f] = f_1 - f_0$ is the jump of the function f at the wave front. Conditions (2.4) and (2.5) imply that, for a discontinuous wave moving over a stationary background ($u_0 = 0$), the equation of the modified shock s-adiabat and the propagation velocity of the wave front are given by the formulas

$$u = u_s(h, h_0, \gamma) = (h - h_0)\sqrt{g(\bar{h} + \gamma)/(hh_0 + \gamma\bar{h})}\ , \tag{2.6}$$

$$D = D(h, h_0, \gamma) = h\sqrt{g(\bar{h} + \gamma)/(hh_0 + \gamma\bar{h})}, \tag{2.7}$$

where $\bar{h} = (h + h_0)/2$ and $h > h_0$. For $h_0 = 0$, these formulas become the finite relation

$$u = D = \sqrt{gh(h + 2\gamma)/\gamma} = \sqrt{\theta gh}, \tag{2.8}$$

where

$$\theta = u^2/(gh) = 2 + h/\gamma \tag{2.9}$$

is the Froude number behind the discontinuous-wave front propagating in a dry channel.

By constructing the classical self-similar solution of the dam-break problem above a horizontal bottom for the modified system (2.1), (2.2), i.e., the Cauchy problem with the initial data

$$h(x, 0) = \begin{cases} H, & x < -l_2, \\ h_0, & x > -l_2, \end{cases} \qquad u(x, 0) = 0, \tag{2.10}$$

where $h_0 = z_0$, we obtain a discontinuous wave propagating at a constant velocity $D = D(h_1, h_0, \gamma) > 0$, a centered depression wave, and a constant-flow region $h_1 = z_1$, $u_1 > 0$ between them (curve 3 in Fig. 1). The constant-flow parameters are found as the coordinates (h_1, u_1) of the point of intersection of the monotonically increasing shock s-adiabat (2.6) and the monotonically decreasing wave r-adiabat:

$$u = v_r(h, H) = 2\sqrt{g}\,(\sqrt{H} - \sqrt{h}\,), \qquad h < H.$$

In particular, for $h_0 = 0$, in view of (2.8), the parameters h_1 and u_1 are related by

$$u_1 = \sqrt{\theta g h_1} = 2\sqrt{g}\,(\sqrt{H} - \sqrt{h_1}\,),$$

which implies that $h_1 = 4H/(\sqrt{\theta} + 2)^2$. From this, using (2.9), we obtain the formula

$$\gamma^* = \frac{\gamma}{H} = \frac{h_1}{(\theta - 2)H} = \frac{4}{(\theta - 2)(\sqrt{\theta} + 2)^2}, \tag{2.11}$$

where the dimensionless parameter γ^* is expressed in terms of the Froude number θ.

From the results of experimental modeling [23] of the dam-break problem (2.10), it follows that, in the case of a dry channel in the tailwater region, i.e. at $h_0 = 0$, the characteristic Froude number behind the discontinuous-wave front is given by the equality $\theta = \theta_1 = 6.7$, whence, by virtue of (2.11), we obtain $\gamma^* = \gamma_1^* = 0.05 \Rightarrow \gamma = \gamma_1 = \gamma_1^* H = 1.03$. This value of γ is used in formulas (2.6) and (2.7) to calculate the flow parameters at the front of the initial discontinuous wave resulting from a dam break.

Figure 2 shows the dependence $h_1^*(h_0^*)$ ($h^* = h/H$ is the dimensionless depth) obtained by solving problem (2.10) for the classical system (2.1), (2.3) (solid curve) and the modified system (2.1), (2.2) for $\gamma^* = \gamma_1^* = 0.05$ (dashed curve). It is evident that significant differences between the classical and modified solutions occur only in the case $h_0^* < h_c^* \approx 0.14$, where the flow (h_1, u_1) behind the discontinuous-wave front is supercritical [12]. This explains why the solutions obtained for the classical system (2.1), (2.3) adequately reproduce the experimental flow pattern in the case where the initial tailwater and headwater depths satisfy the inequality $h_0 > h_c^* H \approx 0.14H$, because of which the constant flow (h_1, u_1) formed behind the discontinuous-wave front is subcritical [1, 21, 23].

To construct a solution for $t > t_0 = l_2/D$, i.e., after the passage of the initial discontinuous waves over the step, for system (2.1), (2.2) one needs to solve the problem of discontinuity decay

$$z(x, t_0) = \begin{cases} z_1, & x \leqslant 0, \\ z_0^*, & x > 0, \end{cases} \qquad u(x, t_0) = \begin{cases} u_1, & x \leqslant 0, \\ 0, & x > 0 \end{cases} \tag{2.12}$$

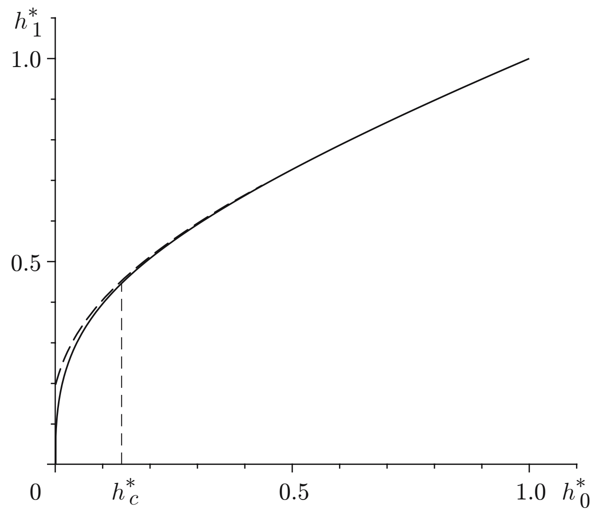

Fig. 2. Relative depths behind the discontinuous-wave front obtained by solving the dam-break problem for the classical (solid curve) and modified (dashed curve) basic conservation laws of shallow-water theory.

above a change in the bed level

$$b(x) = \begin{cases} 0, & x \leqslant 0, \\ \delta, & x > 0, \end{cases} \qquad \delta > 0, \tag{2.13}$$

where $z_0^* = \delta$ for $z_0 \leqslant \delta$ and $z_0^* = z_0$ for $z_0 \geqslant \delta$. The points with the coordinates $z_1 = h_1$ and u_1 [see formula (2.12)] are on the shock adiabat (2.6), i.e.,

$$u_1 = u_s(z_1, z_0, \gamma_1). \tag{2.14}$$

Following [11, 12], we seek a solution for system (2.1), (2.2) of the generalized discontinuity-decay problem (2.12)–(2.14) in the form of a combination of simple waves, a stationary jump located at the coordinate origin above the bottom step, and the constant-flow zones connecting them.

3. Self-Similar Solutions of the Discontinuity-Decay Problem above a Bed Level Change. To construct self-similar solutions of the generalized discontinuity-decay problem (2.12)–(2.14), it is necessary to specify constraints on the flow parameters at the discontinuity formed above the bottom step (2.13). Following [14, 15], we assume that if two characteristics arrive at this discontinuity, then, along with the discharge continuity

$$[q] = 0 \quad \Rightarrow \quad q_1 = q_0, \tag{3.1}$$

which follows for $D = 0$ from the Hugoniot condition (2.4) for the mass conservation law (2.1), it is necessary to require continuity of the Bernoulli function

$$[u^2/2 + gz] = 0 \quad \Rightarrow \quad (u_1^2 - u_0^2)/2 + g(z_1 - z_0) = 0, \tag{3.2}$$

which follows for $D = 0$ from the Hugoniot condition for the local-momentum conservation law.

The solution of the generalized discontinuity-decay problem (2.12)–(2.14) based on the modified system of the basic conservation laws (2.1), (2.2) is constructed by the generalized method of adiabats [13]. This is done using the results of a study [27], in which a similar problem for the case $z_0 \geqslant \delta$ (the step is under water at the initial time) is solved using the classical system of the basic conservation laws (2.1), (2.3). In view of these results, it can be shown that only two flow regimes of type A and B (the solid and dashed curves in Fig. 3) can occur in the experiments performed. In the description of these flows based on the shallow-water equations, two characteristics arrive at the discontinuity above the bottom step, because of which relations (3.1) and (3.2) should hold at this discontinuity.

The self-similar type A solution shown by the solid curve in Fig. 3 consists of a discontinuous wave propagating at a velocity $D_1 > 0$ behind the step, a reflected discontinuous wave propagating at a velocity $D_2 < 0$ ahead

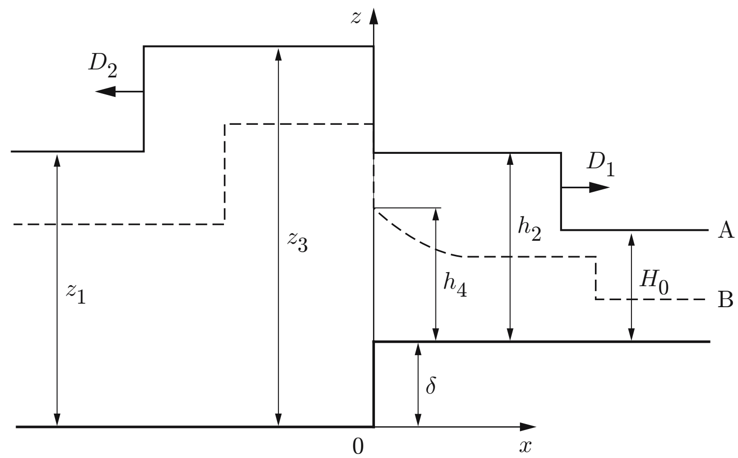

Fig. 3. Theoretical wave profiles resulting form the propagation of the initial discontinuous wave over the bottom step.

of the step, a stationary jump above the step, and constant-flow zones between them (h_2, u_2) and (z_3, u_3). The self-similar type B solution shown by the dashed curve in Fig. 3 differs from the type A solution only in that the constant flow (h_2, u_2) behind the step is continuously transformed to a centered depression r-wave, on whose left boundary the critical flow (h_4, u_4) forms. If $z_0 \leqslant \delta$, i.e., if the step is above water at the initial time, the type B solution holds.

It was found in the experiments that, for $z_0 \leqslant \delta$, the Froude number behind the discontinuous-wave front propagating behind the step in a dry channel depends weakly on the parameter z_0 and that its characteristic value is defined by the equality $\theta = \theta_2 = 15.4$. From an analysis of the type B solution, it follows that this Froude number θ_2 is reached for $\gamma = \gamma_2 = 0.135$. This value of γ is used in formulas (2.6) and (2.7) to obtain the flow parameters at the discontinuous-wave front propagating behind the step. Because the discontinuous wave reflected from the step propagates in a channel of finite depth $h_1 > 10$ cm, it is adequately described by the classical Hugoniot conditions obtained from formulas (2.4) and (2.5) for $\gamma = 0$.

In view of the above assumptions, the constant-flow parameters (h_2, u_2) and (z_3, u_3) in the type A solution are found from the system of equations

$$u_2 = u_s(h_2, H_0, \gamma_2), \qquad u_3 = u_r(z_3, z_1, u_1),$$
$$J(z_3, q) = J(h_2, q) + \delta, \qquad q = h_2 u_2 = z_3 u_3, \tag{3.3}$$

where $u_s(h, H_0, \gamma)$ is the modified shock s-adiabat (2.6) and

$$u = u_r(z, z_1, u_1) = u_1 - (z - z_1)\sqrt{g(z + z_1)/(2zz_1)} \qquad (z > z_1)$$

is the classical shock r-adiabat [3] issuing from the point (z_1, u_1); $J(h, q) = q^2/(2gh^2) + h$. Having determined the parameters h_2, u_2, $z_3 = h_3$, and u_3 from system (3.3), we calculate the propagation velocities of the discontinuous waves D_1 and D_2 by the formulas

$$D_1 = D(h_2, H_0, \gamma_2) = h_2 \sqrt{\frac{g((1 + 2\gamma_2)h_2 + (1 - 2\gamma_2)H_0)}{2h_2 H_0 + \gamma_2(h_2^2 - H_0^2)}},$$
$$D_2 = D_2(h_3, h_1, u_1) = u_1 - \sqrt{gh_3(h_3 + h_1)/(2h_1)}\,. \tag{3.4}$$

In the type B solution, the flow depth and velocity in the centered depression r-wave behind the step (dashed curve in Fig. 3) are determined from the formulas [3]

$$h = (\xi - s)^2/(9g), \qquad u = (2\xi + s)/3, \qquad \xi = x/t \in [0, u_2 - c_2], \tag{3.5}$$

where $s = u_2 + 2c_2$ is the constant value of the s-invariant in the depression r-wave and $c_2 = \sqrt{gh_2}$ is the propagation velocity of small perturbations (sound velocity) in the constant-flow zone (h_2, u_2). On the left boundary of the depression wave (3.5) located on the step, because the flow is critical and $u_4 = c_4 = \sqrt{gh_4}$, the flow parameters (h_4, u_4) are found from the system

$$u_3 = u_r(z_3, z_1, u_1), \qquad J(z_3, q) = J(h_4, q) + \delta, \qquad q = z_3 u_3 = \sqrt{gh_4^3} \tag{3.6}$$

simultaneously with the constant-flow parameters (z_3, u_3) behind the reflected discontinuous-wave front.

Once the parameters h_4 and u_4 are determined, the constant-flow parameters (h_2, u_2) between the discontinuous wave propagating behind the step and the depression r-wave (3.5) are calculated from the system

$$u_2 = u_s(h_2, H_0, \gamma_2) = v_r(h_2, h_4, u_4), \tag{3.7}$$

where $u_s(h, H_0, \gamma)$ is the modified shock s-adiabat (2.6);

$$u = v_r(h, h_4, u_4) = u_4 + 2\sqrt{g}\,(\sqrt{h_4} - \sqrt{h}\,) \qquad (h < h_4)$$

is the wave r-adiabat issuing from the point (h_4, u_4) located on the critical-flow line $u = \sqrt{gh}$. System (3.7) is obtained by solving the shallow-water equations of the classical problem of discontinuity decay above a horizontal bottom [3] with the following initial data:

$$h(x, t_0) = \begin{cases} h_4, & x \leqslant 0, \\ H_0, & x > 0, \end{cases} \qquad u(x, t_0) = \begin{cases} u_4, & x \leqslant 0, \\ 0, & x > 0. \end{cases}$$

Once the parameters h_2, u_2, $z_3 = h_3$, u_3, h_4, and u_4 are determined from systems (3.6) and (3.7), the propagation velocities of the discontinuous waves D_1 and D_2, as for the type A solution, can be found from formulas (3.4).

The intermediate solution separating type A and B flows is a type A solution in which the constant flow (h_2, u_2) behind the step is critical. The parameters $z_0^* = h_0^* = H_0^* + \delta$, z_1^*, u_1^*, h_2^*, u_2^*, z_3^*, and u_3^* of this intermediate solution, which are uniquely determined by the initial headwater depth H and the step height δ, are calculated from the following system of equations:

$$\begin{gathered} u_1^* = u_s(z_1^*, z_0^*, \gamma_1) = v_r(z_1^*, H), \\ u_2^* = \sqrt{gh_2^*} = u_s(h_2^*, z_0^* - \delta, \gamma_2), \qquad u_3^* = u_r(z_3^*, z_1^*, u_1^*), \\ J(z_3^*, q^*) = J(h_2^*, q^*) + \delta, \qquad q^* = h_2^* u_2^* = z_3^* u_3^*. \end{gathered} \tag{3.8}$$

In the experiment, the values $H = 20.5$ cm and $\delta = 5.5$ cm were not changed. Solving system (3.8) for these values of H and δ, we obtain $z_0^* = 7.7$ cm, $H_0^* = 2.2$ cm, $z_1^* = 13.3$ cm, $u_1^* = 55.7$ cm/sec, $h_2^* = 7.1$ cm, $u_2^* = 82.6$ cm/sec, $z_3^* = 15.3$ cm, and $u_3^* = 38.3$ cm/sec. Substitution of these values into formulas (2.7) and (3.4) yields the following values for the velocities of the discontinuous waves in the intermediate solution: $D^* = 133$ cm/sec, $D_1^* = 120$ cm/sec, and $D_2^* = 72$ cm/sec. If the initial tailwater level $z_0 \in [z_{\min}, z_0^*]$, then, after the passage of the initial discontinuous wave over the bottom step, a type B solution takes place; if $z_0 \in (z_0^*, H)$, a type A solution holds. Because $H_0^* > 0$, flows in which a discontinuous wave behind a step propagates in a dry channel $(H_0 = 0)$ are described by type B solutions.

The self-similar type A and B solutions were found by numerically solving the corresponding systems of nonlinear algebraic equations using an iterative method. The accuracy of these numerical solutions is several orders of magnitude higher than the accuracy of the experimental data.

4. Comparison of Theory and Experiment. Figures 4–9 gives results of a comparison of the theoretical and experimental parameters of the flows resulting from the propagation of the initial discontinuous wave over the bottom step.

Time variation of the water free-surface level at a fixed cross section of the channel is shown in Figs. 4 and 5. In the theoretical solution, the main difference between the type A and B waves behind the step is that the free-surface level behind the wave front of type B first reaches a constant value and then increases monotonically, whereas the level behind the type A wave front does not change (see Fig. 3). The experimental data in Fig. 4 are in qualitative agreement with the theoretical result. At the same time, the undulations formed behind the type A wave front are not described by the first approximation of shallow-water theory. The occurrence of the undulations is due to the fact that, on a certain interval behind the type A wave front, the vertical velocities on the liquid

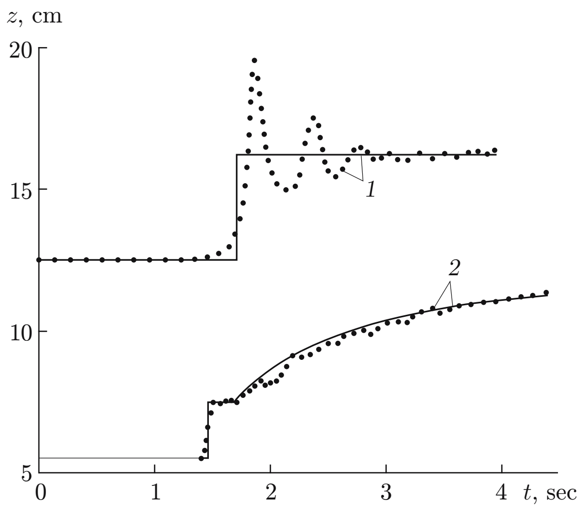

Fig. 4

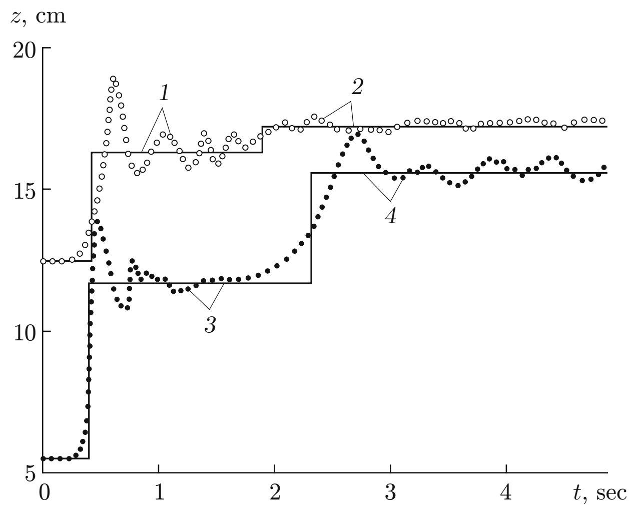

Fig. 5

Fig. 4. Time evolution of the free-surface level behind the step at $x = 80$ cm for type A wave at $z_0 = 12.5$ cm (1) and type B wave at $z_0 = \delta = 5.5$ cm (2): the solid curves refer to the theoretical solution and the points refer to the experimental data.

Fig. 5. Time evolution of the free-surface level ahead of the step at $x = -82$ cm: the solid curve refer to the theoretical solution and the points refer to the experimental data; type A wave incident on the step (1) and reflected from it (2) for $z_0 = 12.5$ cm; type B wave incident on the step (3) and reflected from it (4) for $z_0 = \delta = 5.5$ cm.

surface are comparable to the horizontal velocities. To model these undulations theoretically, it is necessary to use more exact approximations of shallow-water theory or the complete equations of hydrodynamics.

In the theoretical solution, the free-surface level past the discontinuous-wave front behind the step changes suddenly, in particular, in the case of wave propagation in a dry channel (see Figs. 3–5). In experiments, the waves have the form of a moving hydraulic jump. As is known, for stationary flows, five types of hydraulic jump exist, depending on the Froude number [28]. All of them are observed in the nonstationary problem considered. A bore with a roller in the head part and a smooth undular bore are the main types of hydraulic jump. The other types are intermediate.

In Fig. 4, the experimental type A wave is a smooth undular wave, whose first crest broke at a distance $x = 200$ cm from the step. The head of the experimental type B wave propagating in a dry channel is characterized by the presence of a distinct roller. As the initial depth H_0 increases, undulations arise behind the roller, which decrease upstream. From the results of the experiments, it follows that there is a range of x, t, and H_0 in which the waves behind the step have the form of a smooth undular bore. As the initial depth H_0 increases further and the level difference $\Delta z = z_1 - z_0$ becomes sufficiently small, the experimental type A waves behind the step become linear and can be modeled with high accuracy using the linear approximation of shallow-water theory.

The curves shown in Fig. 5 are the profiles of the initial discontinuous wave incident on the step and the discontinuous wave reflected from it. The first sharp increase in the free-surface level occurs when the initial discontinuous wave formed after the removal of the shield above an even bottom reaches the wavemeter and the second occurs when the wave reflected from the step reaches the wavemeter. The experimental type A waves, both the initial wave and the wave reflected from the step, are smooth, and the type B waves are moving hydraulic jumps with the first crest breaking continuously (a bore with a roller [28]).

Figures 6 and 7 show the relative water levels behind the discontinuous-wave fronts propagating behind the step and ahead of it, respectively. The random error in measuring these levels does not exceed the size of the points presented in Figs. 6 and 7. In the case of a smooth undular bore and intermediate types of bore with undulations, the experimental value of the level was taken to be its asymptotic value, i.e., the level value at a large distance from the wave front, where the undulations degenerate. From Figs. 6 and 7, it follows that the theoretical results

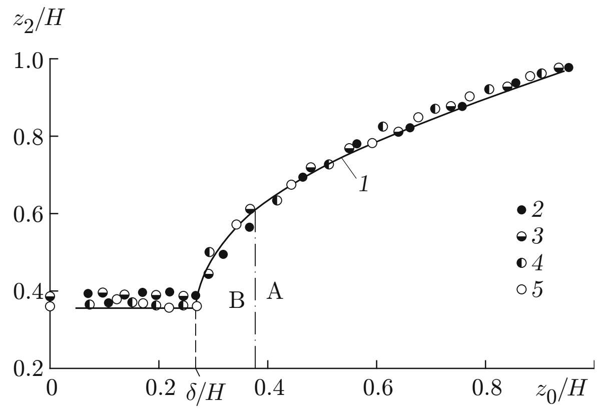

Fig. 6

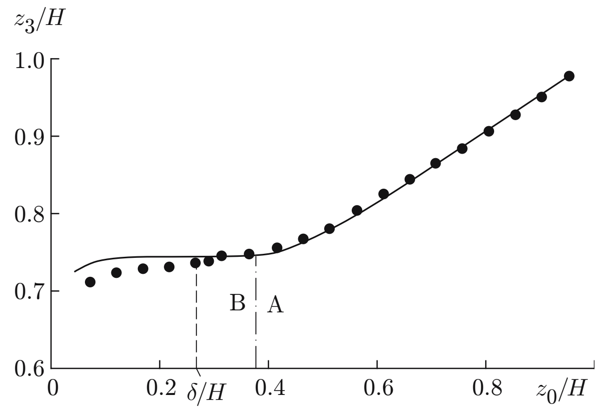

Fig. 7

Fig. 6. Water level behind the wave front propagating behind the step: curve 1 refer to the theoretical solution and points refer to the experimental data at $x = 40$ (2), 80 (3), 120 (4), and 150 cm (5); the dot-and-dashed curve shows the boundary between the regions of existence of type A and B waves.

Fig. 7. Water level behind the wave front reflected from the step: the curve refers to the theoretical solution and the points refer to the experimental data; the dot-and-dashed curve shows the boundary between the regions of existence of type A and B waves.

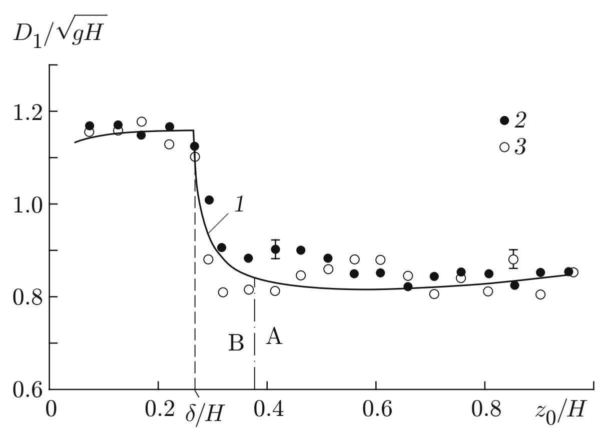

Fig. 8

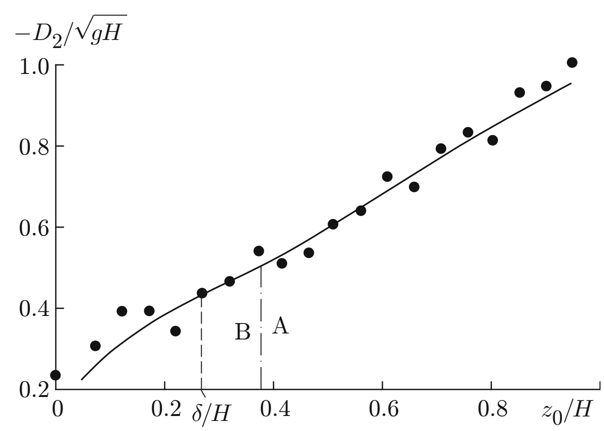

Fig. 9

Fig. 8. Velocity of the discontinuous-wave front behind the step: curve 1 refers to the theoretical solution and points refer to the experimental data (points 2 refer to $x_1 = 45$ cm $x_2 = 75$ cm, and $x = 60$ cm; points 3 refer to $x_1 = 115$ cm, $x_2 = 155$ cm, and $x = 135$ cm); the dot-and-dashed curve shows the boundary between the regions of existence of type A and B waves.

Fig. 9. Velocity of the wave front reflected from the step: the curve refers to the theoretical solution and the points refer to the experimental data ($x_1 = -85$ cm, $x_2 = -55$ cm, and $x = -70$ cm); the dot-and-dashed curve shows the boundary between the regions of existence of type A and B waves.

are in fairly good agreement with the experimental data on the asymptotic depth behind the wave front. The first approximation of shallow-water theory does not describe undulations.

Figures 8 and 9 gives the relative velocities of discontinuous waves behind the step and ahead of it, respectively. In the experiment, this velocity was taken to be the velocity of longitudinal motion of a certain point chosen on the wave profile. Because of breakings at the wave front, the velocity determined in this manner can depend on the choice of the point of the profile. The experimental data in Figs. 8 and 9 correspond to the velocity of motion of the height-averaged point at the wave front. This velocity was determined from the signals of two stationary wavemeters located at the points with the coordinates x_1 and x_2. The indicated velocity was normalized to the value $x = (x_2 - x_1)/2$ corresponding to the middle of the interval $[x_1, x_2]$. For a bore with a roller, its propagation velocity is inevitably measured with a random error due to random changes in the roller shape. Therefore, for some parameter values of the problem, up to five measurement were performed under the same conditions. The standard error obtained in the repeated measurements is shown by vertical segments in Fig. 8. From Figs. 8 and 9, it follows that the theoretical and experimental values of the velocities D_1 and D_2 are in good agreement, in particular, in the case of discontinuous-wave propagation behind the step in a dry channel.

Conclusions. The self-similar solutions constructed in the present work are in fairly good agreement with experimental data on various parameters (types of waves, their propagation velocity, asymptotic depths behind the wave fronts). These solutions were constructed on the bases of shallow-water theory using the modified system of the basic conservation laws describing discontinuous-wave propagation in a dry channel and the assumption of conservation of the total free-stream energy at the stationary hydraulic jump above a bottom step. It should be noted that the mathematical modeling of wave propagation in a dry channel using the complete equations of hydrodynamics is a difficult problem (see [29]). In the solutions of this problem obtained by numerical methods based on the two-dimensional Euler and Navier–Stokes equations (ignoring the entrainment of air bubbles in the wave head), the propagation velocity of the leading edge of the wave is significantly overestimated. In the calculations of the problem of dam break above a horizontal bottom with a dry channel in the tailwater region [29], this velocity asymptotically reaches the propagation velocity of the corresponding depression wave obtained from the classical system of the basic conservation laws of shallow-water theory (2.1) and (2.3). In addition, the indicated numerical solutions can significantly distort the head profile of the wave propagating in a dry channel. At the same time, the results of the present work show that the method proposed here allows the propagation of such waves to be modeled adequately in terms of shallow-water theory by constructing corresponding self-similar solutions (see Figs. 4, 6, and 8).

Thus, the proposed modification of shallow-water theory provides generally adequate descriptions of the wave flows resulting from discontinuous-wave propagation over a bottom step. At the same time, this theory does not describe undulations since it uses a hydrostatic pressure distribution. To describe the undulations, it is necessary to employ models that take into account the deviation from this law, for example, the classical Boussinesq equation or other equations derived from higher-order approximations of shallow-water theory.

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